When a heuristic is consistent it follows the triangular inequality as below.

H(s2) <= cost to reach s2 from s1 + H(s1).

In Gridworld the only possible directions from any node is up, down, right & left.

Let’s assume if the manhattan values are not consistent then, the above inequality becomes.

H(s2)>= cost to reach s2 from s1 + H(s1)

H(s2)-H(s1)>=cost to reach s2 from s1 ------------------------------- (1)

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | H(s1) |  |
| H(s2) |  |  |

Lets say S2 is at x1,y1 and S2 at x2,y2

Manhattan distances between those two is (|x2-x1|+|y2-y1|)

From (1)

(|x2-x1|+|y2-y1|) >= cost to reach s2 from s1

This can be true only when we choose a path that is not a straight line between s2 and s1, for example, can be a diagonal but in our grid world only right, left, up and down are possible.

Therefore the assumption that Manhattan values are not consistent is false.